First, remember that $\frac{1}{2}erfc\left(\sqrt{\gamma}\right) = Q\left(\sqrt{2\gamma}\right)$

So the integral becomes $\int_0^\infty Q\left(\sqrt{2\gamma}\right)p(\gamma)d\gamma$

To solve this, use integrate by parts, by definition $\int f(x)g(x)dx = F(x)g(x) - \int F(x)g'(x)$

Which is another way of rewriting the usual $\int udv = uv - \int vdu$

In this case we have $\int f(\gamma)g(\gamma)d\gamma = F(\gamma)g(\gamma) - \int F(\gamma)g'(\gamma)$

Let $f(\gamma) = p(\gamma) = \frac{1}{\left(\frac{E_b}{N_0}\right)}e^{-\frac{\gamma}{\left(\frac{E_b}{N_0}\right)}}$

$$F(\gamma) = \int f(\gamma)d\gamma = \int \frac{1}{\left(\frac{E_b}{N_0}\right)}e^{-\frac{\gamma}{\left(\frac{E_b}{N_0}\right)}}d\gamma = \frac{1}{\left(\frac{E_b}{N_0}\right)}\int e^{-\frac{\gamma}{\left(\frac{E_b}{N_0}\right)}}d\gamma$$

Then

$$F(\gamma) = \frac{1}{\left(\frac{E_b}{N_0}\right)}\left[-\frac{E_b}{N_0}e^{-\frac{\gamma}{\left(\frac{E_b}{N_0}\right)}}\right] = -e^{-\frac{\gamma}{\left(\frac{E_b}{N_0}\right)}}$$

Let $g(\gamma) = Q\left(\sqrt{2\gamma}\right)$

By definition we have $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}dt$

Also $\frac{d}{dx}\int_{u(x)}^{v(x)} f(t)dt = f\left(v(x)\right)v'(x) - f\left(u(x)\right)u'(x)$

Hence $\frac{d}{dx}Q(x) = \frac{d}{dx}\left(\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}dt\right) = -\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$

And then $\frac{d}{d\gamma}Q\left(\sqrt{2\gamma}\right) = \left(-\frac{1}{\sqrt{2\pi}}e^{-\gamma}\right)\left(\frac{1}{\sqrt{2}}\gamma^{-\frac{1}{2}}\right)$

$$P_b = \int f(\gamma)\, g(\gamma)\, d\gamma = F(\gamma)\, g(\gamma) - \int F(\gamma)\, g'(\gamma)$$

So $P_b = -Q\left(\sqrt{2\gamma}\right) e^{-\frac{\gamma}{\left(\frac{E_b}{N_0}\right)}} \Bigg|_{\gamma=0}^{\gamma=\infty} - \int_0^{\infty} e^{-\frac{\gamma}{\left(\frac{E_b}{N_0}\right)}} \frac{1}{2\sqrt{\pi}} e^{-\gamma} \gamma^{-\frac{1}{2}}$

$$P_b = \frac{1}{2} - \frac{1}{2\sqrt{\pi}} \int_0^{\infty} \gamma^{-\frac{1}{2}} e^{-\gamma\left(1+\frac{1}{\left(\frac{E_b}{N_0}\right)}\right)}$$

To solve the integral, we have $\int_0^{\infty} x^n e^{-ax} = \begin{cases} \dfrac{\Gamma(n+1)}{a^{n+1}} & (n > -1, a > 0) \\ \dfrac{n!}{a^{n+1}} & (n = 0, 1, 2, \ldots, a > 0) \end{cases}$

in this case, use the first part by setting n = -1/2 and x = γ

Hence, we have $P_b = \frac{1}{2} - \frac{1}{2\sqrt{\pi}} \int_0^{\infty} \gamma^{-\frac{1}{2}} e^{-\gamma\left(1+\frac{1}{\left(\frac{E_b}{N_0}\right)}\right)} = \frac{1}{2} - \frac{1}{2\sqrt{\pi}} \left(\frac{\Gamma\left(-\frac{1}{2}+1\right)}{\left(1+\frac{1}{\left(\frac{E_b}{N_0}\right)}\right)^{-\frac{1}{2}+1}} \right)$

By definition, $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

And thus $P_b = \frac{1}{2} - \frac{1}{2\sqrt{\pi}} \left(\frac{\sqrt{\pi}}{\left(1+\frac{1}{\left(\frac{E_b}{N_0}\right)}\right)^{\frac{1}{2}}} \right) = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{1}{1+\frac{1}{\left(\frac{E_b}{N_0}\right)}}} = \frac{1}{2}\left(1 - \sqrt{\frac{\left(\frac{E_b}{N_0}\right)}{1+\left(\frac{E_b}{N_0}\right)}}\right)$